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ROLLING CONTACT FORCE ENERGY RECONSTRUCTION†

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Knowledge of the forces at the wheel–rail contact is fundamental to estimate the consequences in terms of noise and vibration. The traditional use of strain gauges mounted on the wheel web and axle is not capable of determining the high-frequency content of the contact force. Measurements made on the rail are characterized by the spatial variability of input–output transfer functions which makes it difficult to estimate the contact force by simple inversion of the point frequency response function. In this study the problem of rolling contact force reconstruction has been approached through the following steps: (i) the track has been characterized precisely for a finite length by the analysis of the time series of several impacts supplied with an instrumented hammer by using an ARMAX model that proved to be capable of modelling the vertical dynamics of the rail up to 5 kHz; (ii) the response of the rail has been simulated with a random force acting on the system, and the variability of the transfer function has been taken into account by distributing the force on adjacent elements; (iii) the simulated response has been compared with the rail acceleration measured for the passage of several trains; (iv) the wheel–rail contact force has been estimated with a closed-loop algorithm. It has thus been possible to reconstruct the $\frac{1}{3}$ octave power spectrum of contact forces with a simple and stable iterative procedure. Forces reconstructed from different sensors were found to be practically the same for a given wheel; forces from nominally similar wheels are statistically examined and partial results of comparisons made on different rolling stock are shown.

1. INTRODUCTION

In many situations the direct measurement of the dynamic forces acting on a system cannot be performed with traditional procedures and transducers; typical examples are the forces distributed over a system, the inaccessibility of the point of application and the mobility of the force. In these cases it is not even possible to use a force transducer and the estimation of the force, or at least of its magnitude, can be made only through indirect measurement techniques.

A typical problem which is particularly complex and not yet completely solved, is the estimation of the wheel–rail contact force. Rolling forces are the main determinants of vibration in both the wheels and the rails that radiate noise in the environment. The wheel and the track are the most important noise sources, (dominating pantograph, running gear and aerodynamic noise) in the speed range from 50 to 250 km/h, and their frequency distribution shows notably high components up to around 5 kHz. For this reason it is

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particularly relevant to determine rolling contact forces at high frequencies, and both the measurement chain and the reconstruction algorithm must be absolutely reliable.

Contact force reconstruction can be performed by using output (acceleration) signals measured on the rolling wheel or on the rail. In both cases there are some difficulties; for the accelerometers mounted on the wheel the signal output can be considered quasi-stationary, at least for small time intervals, but the instrumentation is particularly expensive since a telemetry system with a high specification is required; for the rail the phenomenon is typically transient, and this complicates the experimental data processing to reconstruct the force. In spite of this, the second way has been preferred not only for economic reasons (as it requires only "static" instrumentation) but also because its application is more useful, as it provides a comparison of contact forces due to different wheels on the same train and for different rolling stock. Moreover, it represents a simple and cost-effective approach to monitoring wheel-tread conditions.

The authors have developed a completely general algorithm for the high-frequency reconstruction of a mobile excitation by using the system properties and the output under a moving input [1]; in this paper a faster, simplified and particularly stable procedure for the reconstruction of the $\frac{1}{3}$ octave band spectrum of the contact force is presented.

2. SYSTEM IDENTIFICATION

Railway tracks have some peculiarities that make their modelling almost impossible with the techniques traditionally used in mechanical systems analysis:

1. rails are "infinitely" long and so they behave, relative to vibration propagation, as waveguides;
2. rail cross-section undergoes deformations in its plane at frequencies above around 1 kHz and so rail modelling as an Euler–Bernoulli or a Timoshenko beam is not effective;
3. sleepers act as discrete supports and at the so-called *pinned–pinned resonance* they coincide with the nodes of the longitudinal deformation;
4. the global damping of the system is very high, as the rail pads dissipate energy and the ground below the sleepers behaves like a damper; the attenuation along the rail ranges from 1 to 5 dB/m depending on the frequency;
5. some elements, like the ballast and the rail pads, can only be approximated as a lumped spring–damper pair.

Mechanical systems modelling is usually carried out by using lumped parameters models (mass–spring–damper) or non-parametric models through the input–output relationships in the frequency domain (frequency response function—FRF) or in the time domain (impulse response function—IRF).

The first approach, known as the modal approach, has the great advantage that it provides values that are directly related to the physical properties of the system, but it is not applicable to all structures. If the system has highly coupled modes, high damping or numerous local modes (typically at the higher frequencies) it is impossible to extract modal parameters. This method is not applicable at all to railway rails, as they have waves rather than modes (they are infinite in nature, and eigenmodes arise only for boundary reflections).

The second approach is general and proves to be particularly efficient for modelling the output response in any measured degree of freedom for any input (real or simulated) applied to the system. Nonetheless, the input reconstruction from output data in the frequency domain is almost impossible because of ill-conditioning problems during the

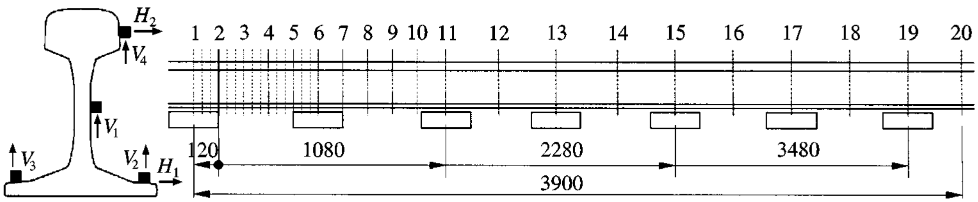

Figure 1. Left: vertical (V) and horizontal (H) accelerometers location in the instrumented section #2. Right: analyzed track section (only numbered sections have been used for model construction) (rail lengths are in mm).

FRFs matrix inversion; moreover, this approach shows other limits in the time domain (a windowing is necessary to reduce leakage errors) when the time series have variable lengths that are not multiples of two (the use of discrete Fourier transform is required instead of fast Fourier transform).

For the representation of the dynamic behaviour of the track an Autoregressive Model (ARMAX) has been set-up (Figure 1). The main properties of this model is briefly summarized here (for a detailed description of the model and of its set-up see reference [2]):

- the model is parametric, i.e., it describes the behaviour of the system with a finite number of parameters which are not related to the physical properties of the system (mass, damping, stiffness) but which are only a measure of how input signals are "transformed" to output signals;
- the parameters are extracted by using overdetermined information, ensuring the highest reliability of the estimation; moreover, it is possible to measure the parameters' indetermination and, with appropriate model validation techniques, to reduce it;
- it is possible to estimate the effect on the output of non-measured inputs (noise); if this effect is small it means that the output only depends on the input and that it is possible to estimate the inputs from the (known) output, otherwise, a non-negligible contribution would be ascribed to the unknown input when inverting the noise inevitably present in the output;
- the number of the parameters that describe the system is small; the model is more stable than FRF-based models where each spectral line is estimated independently (high local errors can result even in models derived from very accurate measurements and processing), performing a kind of "smoothing" of FRFs, thereby neglecting local errors.

Experimental data for the model set-up were collected during a specific test programme carried out in January 1996 on the Firenze–Roma line, close to the main Firenze Santa Maria Novella station, on a track with UIC 60 rails mounted on concrete sleepers, on a ballast bed; the accelerometers, mounted on the section for track characterization, were also used to collect measurements for pass bys of around 70 passenger trains of different composition. The track model is able to represent the dynamic behaviour of a span of several metres of the railway line with high resolution up to 5 kHz.

3. CONTACT FORCE RECONSTRUCTION

Many authors have developed analytical and experimental procedures to estimate rolling contact forces from the measurement of the output, i.e., the acceleration measured on the rail. For example, in 1983 ten Wolde and Van Ruiten [3] published their results obtained by using the point rail FRF and considering a time interval centred on the instant when the wheel passes over the instrumented section. The contact force frequency spectrum was estimated by the product of the inverted point FRF and the output frequency spectrum. This technique, also used by one of the authors in a previous work [4], does not take any account of the contact forces before and after the passage over the instrumented section except for those in close proximity where the FRF changes considerably. Data in reference [3] are particularly useful for the frequency range analyzed and for the rolling stock considered, as they are a basis for the evaluation of the quality of the results found with the new approach described here.

In this research both the mobility of the excitation and the spatial variability of the transfer function of the system are considered. Due to the very high damping of the system, only one wheel is considered at any time. This hypothesis can be considered valid if the wheel whose force is to be reconstructed is carefully chosen, i.e., by selecting only the inner wheelsets (wheelsets #2 and #3 on normal 2-bogey coaches), with a bogey-minimum overestimation of the force due to the contribution of adjacent non-considered wheels on rail acceleration.

The reconstruction algorithm developed uses an ARMAX model of the track to simulate the response of the system to a known force that moves with the same law of motion as the wheel considered. By comparing the simulated output with the measured response under the real input a correction to the input force that reduces the difference is applied. The hypothesis is that the force that produces a response equal to the real response is equivalent to the real force. It is important to underline that in this approach attention was only focussed on the energy content of forces and accelerations; comparisons are made between $\frac{1}{3}$ octave band frequency spectra. This representation of the signals does not provide any phase information, but this does not represent a limit for the application of the procedure, as the signals considered here are basically random in nature.

The whole procedure requires only the response measured at just one point on the rail. As the measurements were performed at four different points (all of them were used to build up the ARMAX model of the track), it is possible to reconstruct independently four (nominally equal) unknown forces from the signals from the different accelerometers. By comparing these four estimates for the same wheel it is possible to evaluate the reliability of the estimation.

3.1. THE ALGORITHM

The algorithm developed consists of the following steps:

1. a pseudo-random signal with flat frequency spectrum up to 5 kHz is generated several times with variable random phase. The amplitude of this first signal is given by the ratio of the average level of the measured acceleration to the average level of all the measured transfer functions of the system. These signals only roughly represent the wheel–rail contact force. As the real phases of the inputs of the system are unknown, the output estimation is repeated with signals of the same amplitude but with different random phases and the magnitudes are averaged to extract energy information. The output energy estimation proved to be stable after averaging 30 different-phase input forces;
2. a set of 30 responses of the rail during the passage of the wheel is estimated by using the technique described in the next paragraph, assuming the contact force has the trend defined at step 1;
3. the $\frac{1}{3}$ octave band output of the system is evaluated from the set defined at step 2. For this computation only the portion of the signal related to sections 1–14 was used, as the output in this interval is only due to the forces acting on the modelled track span (it

contains energy also from sections 15–20). Acceleration measured on the rail when the wheel passes over elements 15–20 (about 2 m span) contains energy from before the wheel enters the modelled section and it is therefore neglected;

4. the rail acceleration frequency spectrum obtained by simulation is compared with the measured spectrum. The difference between them represents the estimation error. For each $\frac{1}{3}$ octave band a gain, to be applied to the input force, is computed to minimize the difference between real and the estimated responses;
5. 30 signals with the corrected energy frequency content and random phase are applied to the system, iterating the procedure from step 2.

The process is stopped when the error defined at step 4 is within a defined limit. Usually, two iterations proved to be sufficient.

3.2. SIMULATION OF THE TRACK RESPONSE

The system considered here consists of 20 inputs and 4 outputs. When a wheel passes over the modelled track span, each element shown in Figure 1 is subjected to distinct segments of the contact force, with an amplitude that depends on the speed. Under the hypothesis of linearity the principle of superposition holds, and the response of the system can be easily computed as the sum of the outputs due to the application of different forces applied to the different elements.

The excitation of the system through the application of distinct signal segments to rail elements is too variable. To obtain smoother and more realistic results a triangular window has been superimposed to each segment, so that adjacent segments are linearly weighted. The underlying hypothesis is that the FRFs change linearly with space between two adjacent points. For reasons of software implementation a different but almost equivalent approach was used; constant FRFs were assumed for each element, while the force was distributed to act on an element when it was just over its centreline and decreases to zero by moving to the centreline of the adjacent element (Figure 2). These weighted force segments were fed to the corresponding elements of the ARMAX model of the track.

4. SOME EXAMPLES OF FORCE RECONSTRUCTION

The algorithm developed is particularly fast and stable. The results shown here are specific to particular rolling stock but, as almost all types of Italian passenger rolling stock passed over the instrumented section, only selected results are presented. Trains have been selected by using the following criteria:

- only trains of “fixed formation” are analyzed here, ensuring an equal mileage run by all of the wheels;
- only trains at constant speed have been chosen (this is a critical condition as almost all trains accelerate in the measuring site) such that energy analysis is formally correct;
- whenever possible, results are compared for similar rolling stock running at different speeds;
- the ETR450 (8 car EMU, known as the first generation “Pendolino”) and ETR460 (9 car EMU, the new generation “Pendolino”) were chosen as examples of homogeneous light rolling stock for high-speed services (up to 250 km/h);
- the ALe601 (up to 8 vehicles, but typically 4 or 5 vehicles EMU) has some tread-braked motorized elements and some disc-braked trailer coaches; it was developed in the 1960s as a luxury train with very good ride characteristics.

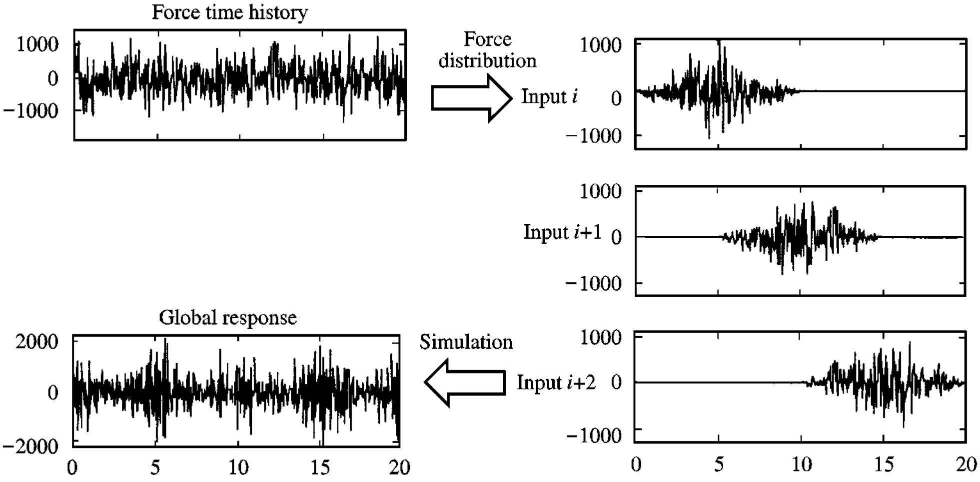

Figure 2. Application of the estimated contact force to the system. The signal is linearly distributed on adjacent elements as explained in the text body.

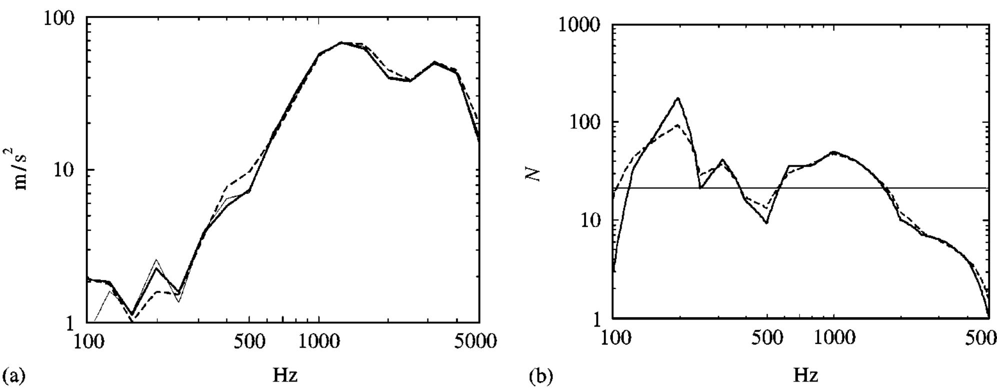

Figure 3. Acceleration in position V1 and reconstructed forces for a wheel of an ETR450 train running at 80 km/h. Measured (thin line) and estimated (1st iteration dashed line, 2nd iteration thick line) $\frac{1}{3}$ octave acceleration (a). Rolling contact forces narrow band spectra: initial pseudo-random force (thin line), 1st iteration (thick dashed line), 2nd iteration (thick continuous line) (b).

Initially, the reconstruction of the rail acceleration in the V1 position is compared with the measured acceleration (Figure 3(a)). The reconstruction process converges with measured results after a few iterations: it is possible to see, after just two iterations, how the results can be considered to be satisfactory (the initial resulting acceleration is meaningless as it is derived from a flat spectrum force and it is not shown). The force required to obtain such an acceleration spectrum is shown in Figure 3(b) where, starting from the constant average level spectrum explained in section 3.1, the final force has a notably different shape.

As previously stated, the presence of four outputs allows an estimation of the statistical reliability of the reconstructed forces to be made. Forces reconstructed for a given wheel by using different accelerometers are very similar, confirming the validity of the proposed procedure. Figure 4(a) shows as an example the force reconstructed from V1 to V4 for an ETR450 wheel; it must be noted that, generally speaking, signals reconstructed by using V4 signals are sometimes quite different from V1 to V3 signals, but this can be due to the substantially different dynamic behaviour of the rail at point V4 and to the transverse

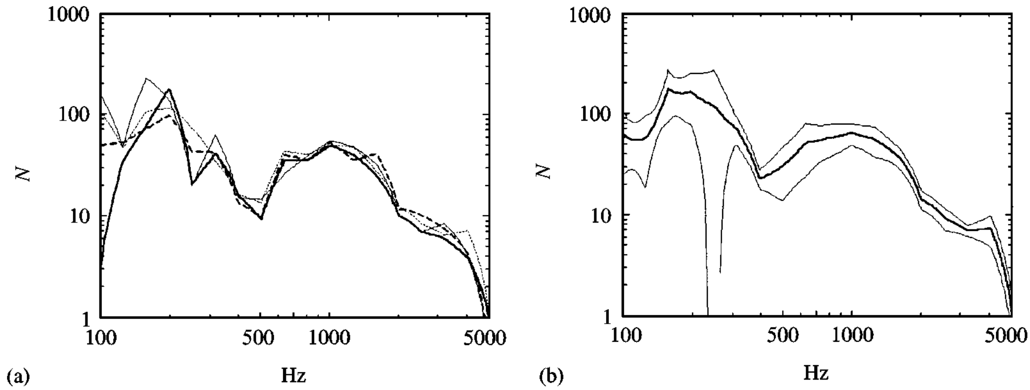

Figure 4. Contact force narrow-band spectrum for a wheel of an ETR450 train running at 80 km/h estimated V1 (thick continuous line), V2 (thick dashed line), V3 (thin continuous line) and V4 (thin dashed line) accelerations (a). Mean value and confidence interval $\pm\sigma$ for all the wheels of the ETR450 train. The force of each wheel is the average of the forces reconstructed by V1–V4 accelerometers (b).

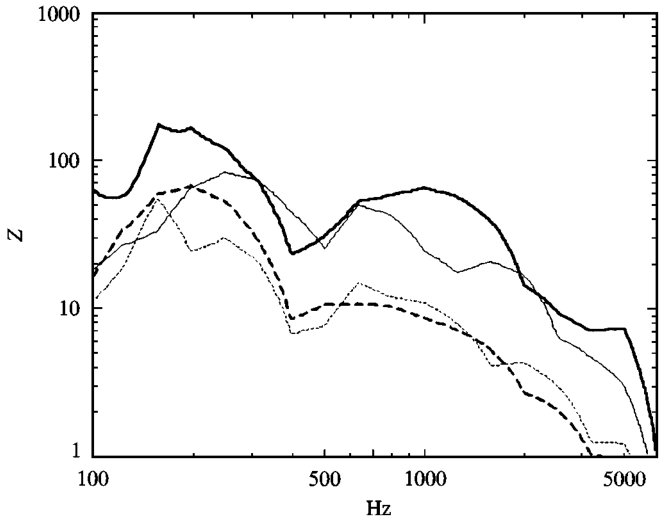

Figure 5. Comparison of average rolling contact forces for an ETR450 train running at 80 km/h (thick continuous line), an ETR460 train running at 40 km/h (thick dashed line), an ALe 601 train running at 55 km/h (thin continuous line: tread braked, thin dashed line: disc braked).

sensitivity of the transducer that influences this measurement. For subsequent analyses the average of the four reconstructed forces was assigned to each wheel. Forces reconstructed for nominally equal wheels are quite similar (Figure 4(b)), as the confidence interval increases only at some frequencies. Reconstructed contact forces for different rolling stock are compared in Figure 5. While the trend remains basically the same, showing some lobes with an overall decrease at the higher frequencies, important differences are evident between tread- and disc-braked rolling stock. Rolling forces increase with speed as expected.

5. CONCLUSIONS AND FURTHER DEVELOPMENT

The results obtained with a new procedure for rolling contact force reconstruction based on time history analysis and on an autoregressive model of the track are particularly promising. Results are stable and plausible, and important differences for different rolling stock are predicted.

The procedure can be usefully employed as an indirect method for wheel-tread diagnostics, for the optimization of wheel tread and web shapes and for the generation of realistic controlled forces in laboratory bench noise emission tests.

An extensive analysis will be performed on the acquired data. Resulting forces are particularly low because of the limited line speed, and a new measurement programme which will include trains running at higher speeds will be necessary. It is expected that the contact force energy content at higher frequencies will increase coherently with speed as does the noise frequency content. It is important to highlight that an increase in the speed, and a corresponding reduction in time history length, does not represent a problem for the procedure for the particular time-based signal analysis.

Development of the contact force reconstruction algorithm presented in this paper is in progress; the modified technique will be capable of simultaneously performing the estimation of the forces corresponding to both wheelsets of a bogie. This extension to a double unknown force reconstruction requires knowledge of the responses corresponding to two-system outputs. In order to minimize the force estimation error, the information content of the outputs must be uncorrelated, i.e., the responses must be independent. The experimental data collected in the measurement programme described in this work do not meet this requirement, since all the responses are measured on the same rail section, albeit at different points. These points have an independent dynamic behaviour only if the cross-section of the rail is deformed, i.e., over 1 kHz.

Further measurements were performed in November 1998 with several instrumented rail sections, in order to validate the modified force reconstruction algorithm, briefly outlined in the following procedure:

1. two sets of pseudo-random signals, corresponding to the unknown inputs, were generated;
2. the responses of the system of the forces defined at step 1, acting separately, were estimated;
3. through the estimation of the $\frac{1}{3}$ octave band outputs of the system, the "influence factors" of each input force were computed, defined as the contribution to the system response due to each wheel acting with a unit force on the modelled track;
4. the contact force reconstruction was performed as described above for a single-input system; the $\frac{1}{3}$ octave band gain, corresponding to each unknown force, was computed through the solution of a linear system of two equations whose unknowns were the gains. The constant vector is represented by the measured responses of the system and the coefficients of the matrix are the influence factors. The system has a unique solution (the matrix can be inverted) as the outputs of the system are independent.

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